

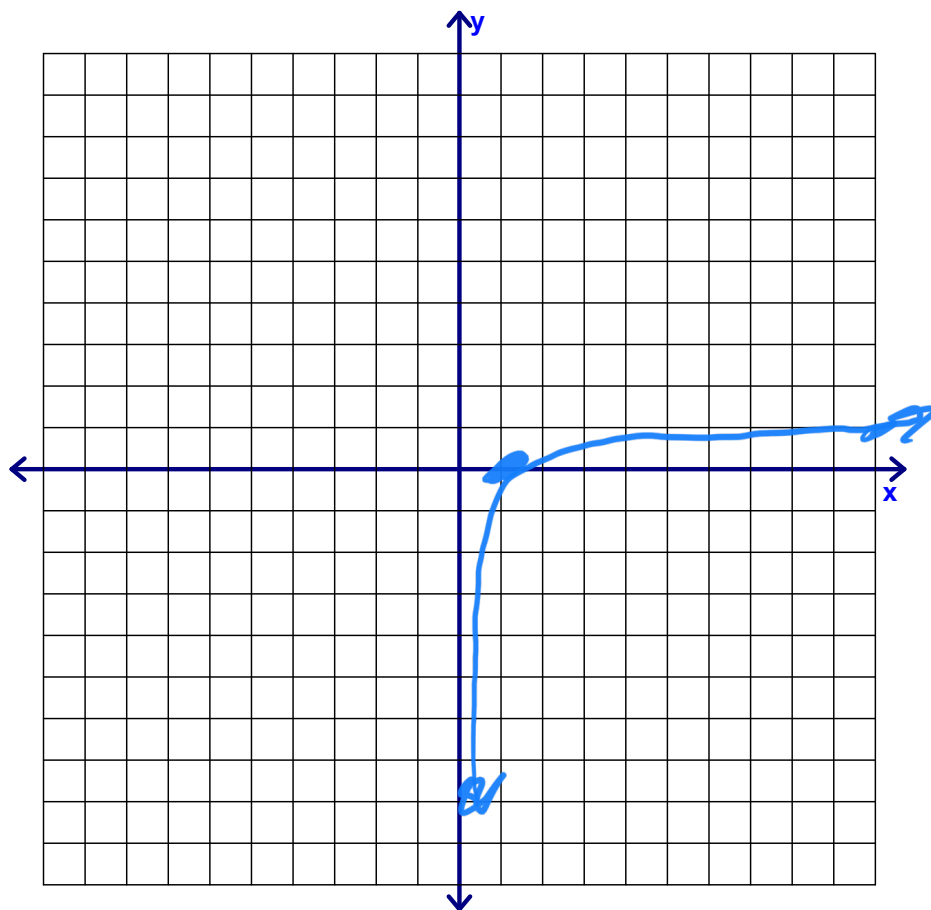
Pre-Calculus CP 1 – Section 3.3 Notes
Properties of Logarithms

Name: Key

Graph the following:

$$y = \log x$$

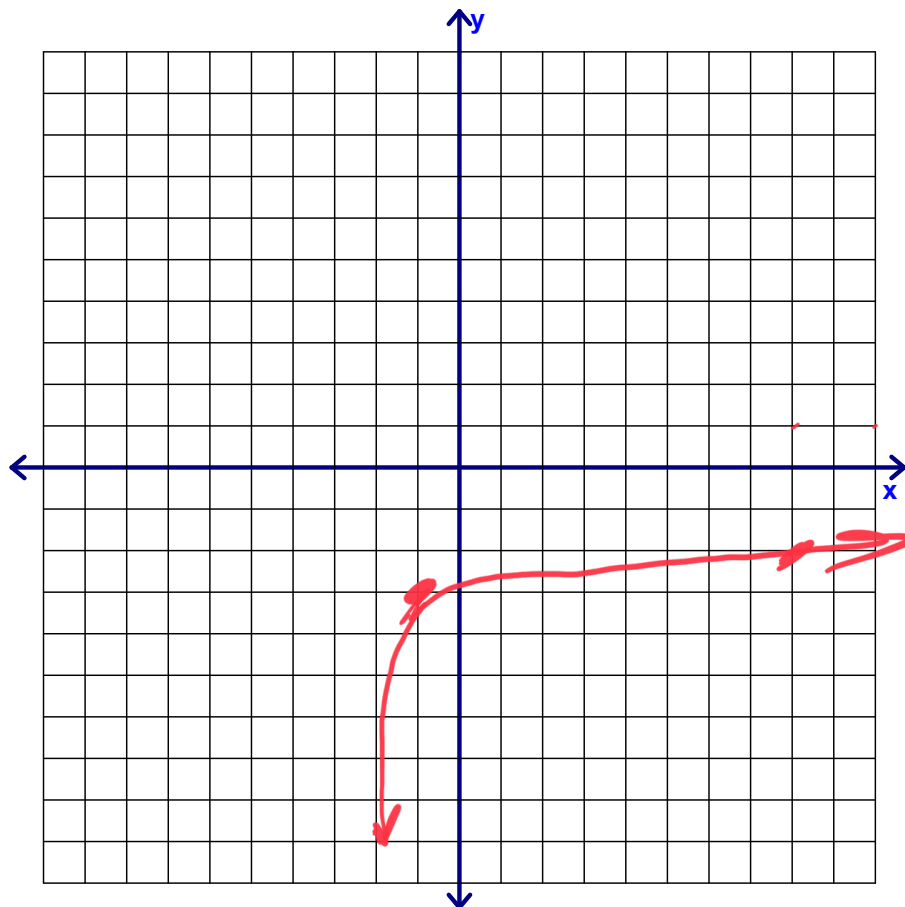
$$y = \log_{10} x$$



Properties of Logarithms

$$y = \log(x+2) - 3$$

① shift left by 2
② shift down by 3

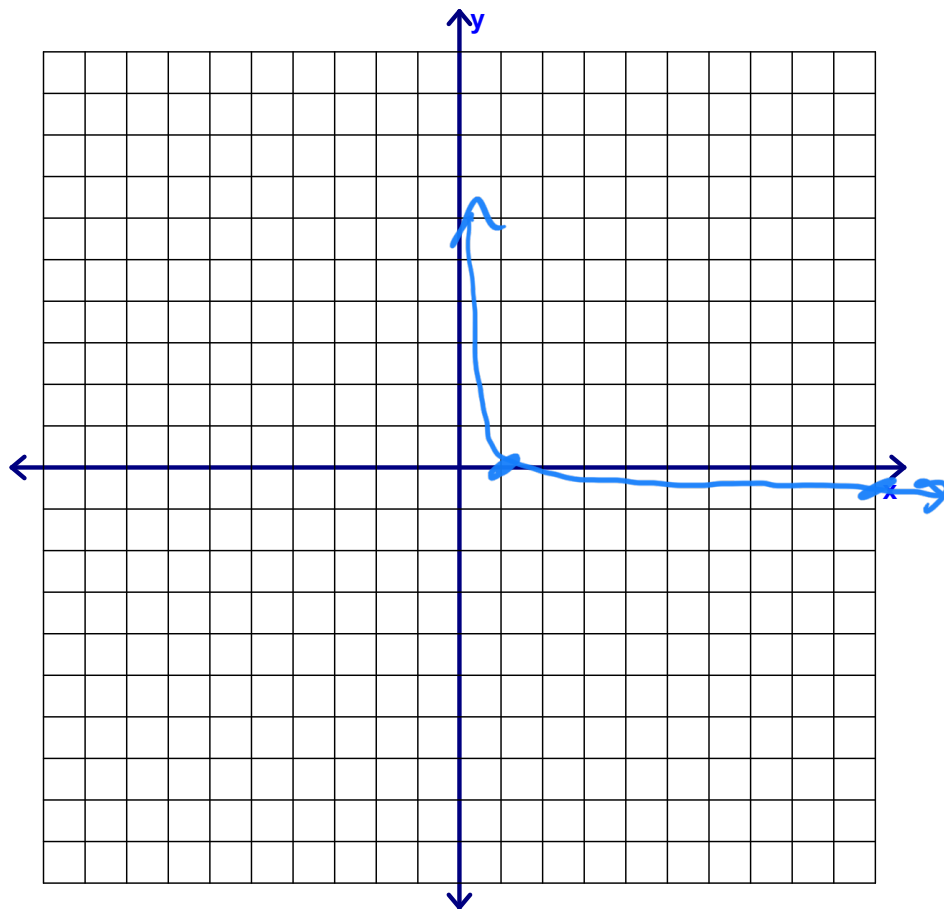


Properties of Logarithms

$$y = -\log\left(\frac{1}{2}x\right)$$

Reflection over
x-axis

horizontal
stretch
by factor of 2



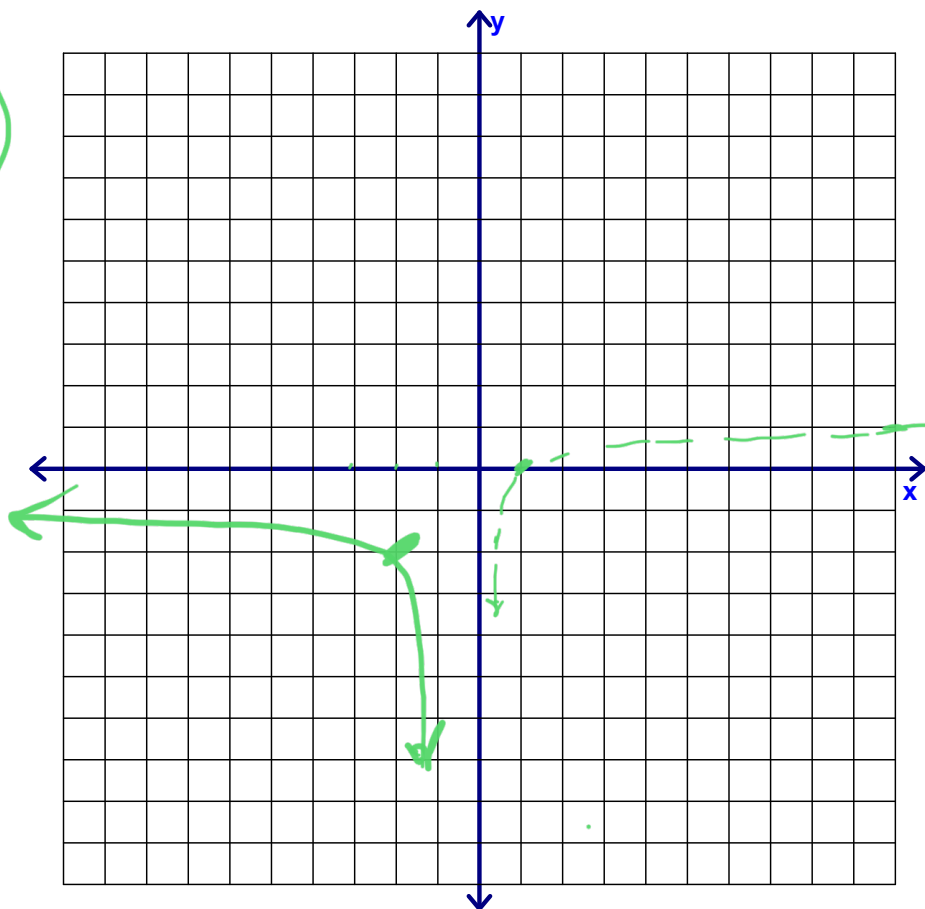
Properties of Logarithms

$$y = \log(-(x+1)) - 2$$

reflect over y-axis

shift left 1 unit

shift down 2 units



Properties of Logarithms

Calculators only have two keys to evaluate logarithms:

- logarithm (LOG) which is base 10
- natural logarithm (LN) which is base “e”

What if you need to evaluate logarithms with other bases?

Change-of-Base Formula

Let a , b , and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. The $\log_a x$ can be converted to a different base as follows:

Base b

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Base 10

$$\log_a x = \frac{\log x}{\log a}$$

Base e

$$\log_a x = \frac{\ln x}{\ln a}$$

Ex 1: Rewrite each logarithm as a ratio of common logarithms and natural logarithms

a) $\log_7 4 = \frac{\log 4}{\log 7} = \frac{\ln 4}{\ln 7}$

2.7124

b) $\log_2 17 = \frac{\log 17}{\log 2} = \frac{\ln 17}{\ln 2}$

4.0675

try them in your calculator to ensure you did it right!

We already learned about how to rewrite logs in exponential form, so these inverse properties should make sense

▪ $\log_b b^x = \underline{x}$

▪ $b^{\log_b x} = \underline{x}$

▪ $\log_b b = \underline{1}$

▪ $\log_b 1 = \underline{0}$

$y = \log_b x$

$b^y = x$

Properties of Logarithms

Ex 2: Use the properties above to evaluate the logarithmic expressions.

a) $3\log_5 5 - \log_5 25 = 3(1) - 2 = 1$

b) $\log_4 16^2 + \log_4 8^4 = \log_4 (4^2)^2 + \log_4 (2^3)^4 = \log_4 4^4 + \log_4 2^{12} = 4 + 6 = 10$

c) $\log_4 2 + \log_4 32 = \frac{1}{2} + \log_4 2^5 = \frac{1}{2} + \log_4 (2^2)^{5/2} = \frac{1}{2} + \log_4 4^{5/2} = \frac{1}{2} + \frac{5}{2} = 3$

More Properties of Logarithms

Let a be positive real number such that $a \neq 1$ and let n be a real number. If u and v are positive real number, the following properties are true:

- 1) $\log_a (u \cdot v) = \log_a u + \log_a v$
"the log of a product equals the sum of the logs"
- 2) $\log_a \left(\frac{u}{v} \right) = \log_a u - \log_a v$
"the log of a quotient equals the difference of the logs"
- 3) $\log_a u^n = n \log_a u$
"the log of a number raised to a power equals the power times the log"

Ex 3: Expand each logarithmic expression:

a) $\log_4 xy = \log_4 x + \log_4 y$

Properties of Logarithms

b) $\log_3 x^4 = 4 \log_3 x$

$$(xy)^3 = x^3 y^3$$

c) $\log_7 xy^3 = \log_7 x + \log_7 y^3$
 $= \log_7 x + 3 \log_7 y$

d) $\log_8 \frac{\sqrt{2x+5}}{7} = \log_8 \sqrt{2x+5} - \log_8 7$
 $= \frac{1}{2} \log_8 (2x+5) - \log_8 7$

You try: Expand the logarithmic expression:

a) $\log_4 x^3 y^2 = \log_4 x^3 + \log_4 y^2$
 $= 3 \log_4 x + 2 \log_4 y$

b) $\log_5 \sqrt{\frac{x}{y^5}} = \frac{1}{2} (\log_5 \frac{x}{y^5}) = \frac{1}{2} (\log_5 x - \log_5 y^5)$
 $= \frac{1}{2} (\log_5 x - 5 \log_5 y) = \frac{1}{2} \log_5 x - \frac{5}{2} \log_5 y$

c) $\log_5 \sqrt[4]{y} = \log_5 y^{\frac{1}{4}} = \frac{1}{4} \log_5 y$

Properties of Logarithms

d) $\log_7\left(\frac{x}{9}\right) = \log_7 x - \log_7 9$

Ex 4: Condense each logarithmic expression.

a) $3\log_3 x + \log_3 y = \log_3 x^3 + \log_3 y$
 $= \log_3 x^3 y$

b) $2\log_4 x + \log_4 3 - \log_4 y = \log_4 x^2 + \log_4 3 - \log_4 y$
 $\log_4 x^2 \cdot 3 - \log_4 y$
 $= \log_4 \frac{3x^2}{y}$

HW: 3.3: p. 243 # 3 - 75 multiples of 3

3, 6, 9, ..., 75